Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

GRAVITATIONAL FIELD

Data given in CAIE paper:

gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
acceleration of free fall $g = 9.81$ m s^{-2}

gravitational potential $\phi = -\dfrac{Gm}{r}$

1 (a) State Newton's law of gravitation.

....

....

....[2]

(b) The planet Jupiter and one of its moons, Io, may be considered to be uniform spheres that are isolated in space.
Jupiter has radius R and mean density ρ.
Io has mass m and is in a circular orbit about Jupiter with radius nR, as illustrated in Fig. 1.1.

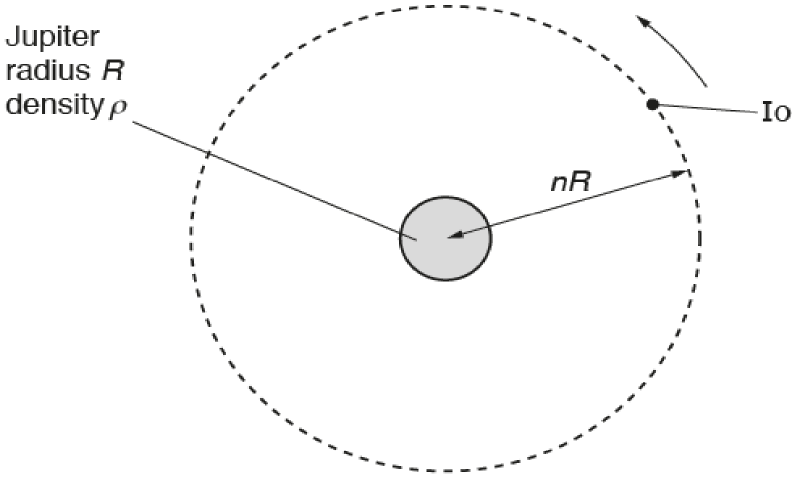

Fig. 1.1

The time for Io to complete one orbit of Jupiter is T.

Show that the time T is related to the mean density ρ of Jupiter by the expression

$$\rho T^2 = \frac{3\pi n^3}{G}$$

where G is the gravitational constant.

[4]

(c) **(i)** The radius R of Jupiter is 7.15×10^4 km and the distance between the centres of Jupiter and Io is 4.32×10^5 km.
The period T of the orbit of Io is 42.5 hours.

Calculate the mean density ρ of Jupiter.

ρ = ... $kg\,m^{-3}$ [3]

(ii) The Earth has a mean density of $5.5 \times 10^3\,kg\,m^{-3}$. It is said to be a planet made of rock. By reference to your answer in **(i)**, comment on the possible composition of Jupiter.

....

....[1]

{N-17/42/Q.1}

2.

(a) Explain how a satellite may be in a circular orbit around a planet.

....

....

....[2]

(b) The Earth and the Moon may be considered to be uniform spheres that are isolated in space. The Earth has radius R and mean density ρ. The Moon, mass m, is in a circular orbit about the Earth with radius nR, as illustrated in Fig. 1.1.

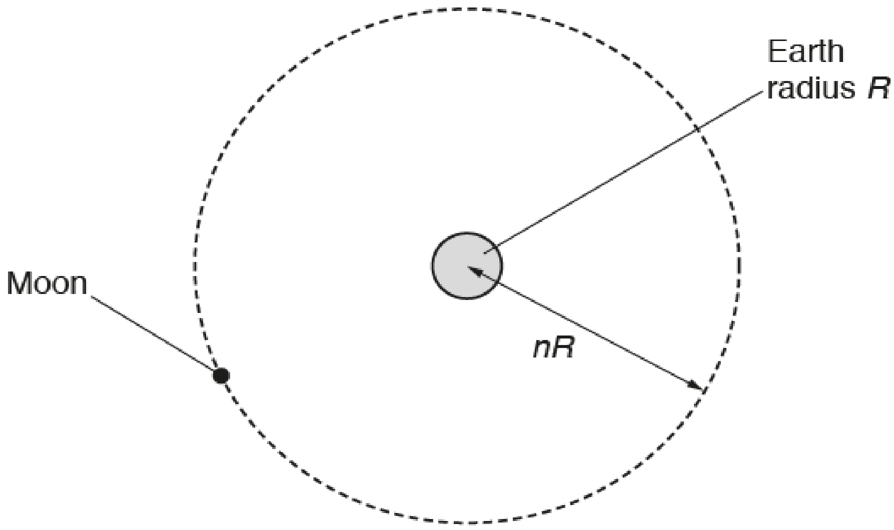

Fig. 1.1

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The Moon makes one complete orbit of the Earth in time *T*.
Show that the mean density ρ of the Earth is given by the expression

$$\rho = \frac{3\pi n^3}{GT^2}.$$

[4]

(c) The radius *R* of the Earth is 6.38×10^3 km and the distance between the centre of the Earth and the centre of the Moon is 3.84×10^5 km.
The period *T* of the orbit of the Moon about the Earth is 27.3 days.
Use the expression in **(b)** to calculate ρ.

ρ = ... kg m^{-3} **[3]**

{*June 17/41/Q.1*}

3.
A satellite is in a circular orbit of radius *r* about the Earth of mass *M*, as illustrated in Fig. 1.1.

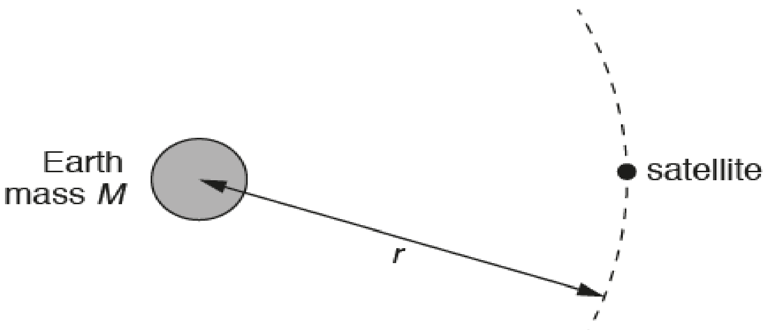

Fig. 1.1

The mass of the Earth may be assumed to be concentrated at its centre.

(a) Show that the period T of the orbit of the satellite is given by the expression

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

where G is the gravitational constant. Explain your working.

[3]

(b) **(i)** A satellite in geostationary orbit appears to remain above the same point on the Earth and has a period of 24 hours.
State two other features of a *geostationary* orbit.

1. ....

....

2. ....

....

[2]

(ii) The mass M of the Earth is 6.0×10^{24} kg.
Use the expression in **(a)** to determine the radius of a geostationary orbit.

radius = ... m [2]

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(c) A global positioning system (GPS) satellite orbits the Earth at a height of 2.0×10^4 km above the Earth's surface.
The radius of the Earth is 6.4×10^3 km.

Use your answer in **(b)(ii)** and the expression

$$T^2 \propto r^3$$

to calculate, in hours, the period of the orbit of this satellite.

period = ... hours [2]

{*Nov. 16/41 & 42/Q.1*}

4.

(a) Define *gravitational field strength.*

....

....[1]

(b) An isolated star has radius *R*. The mass of the star may be considered to be a point mass at the centre of the star.
The gravitational field strength at the surface of the star is g_s.

On Fig. 1.1, sketch a graph to show the variation of the gravitational field strength of the star with distance from its centre. You should consider distances in the range *R* to 4*R*.

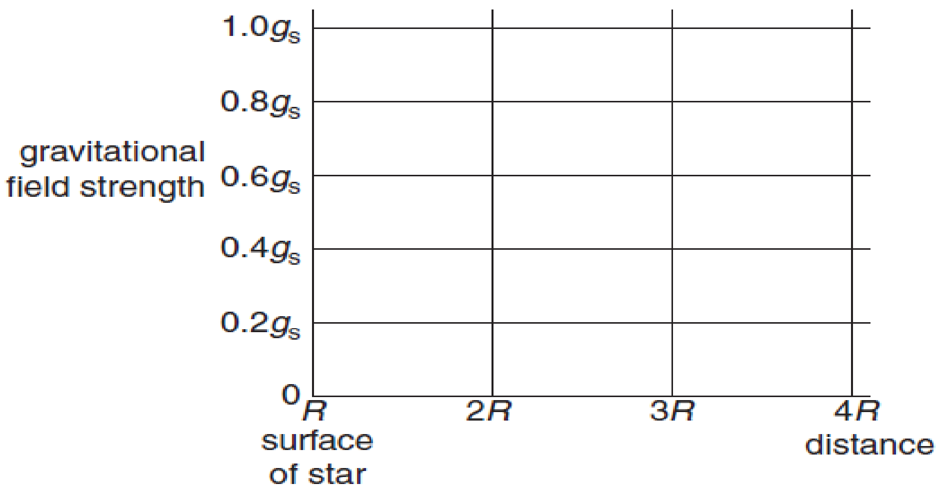

Fig. 1.1

[2]

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M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
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(c) The Earth and the Moon may be considered to be spheres that are isolated in space with their masses concentrated at their centres.
The masses of the Earth and the Moon are 6.00×10^{24} kg and 7.40×10^{22} kg respectively.
The radius of the Earth is R_E and the separation of the centres of the Earth and the Moon is $60\,R_E$, as illustrated in Fig. 1.2.

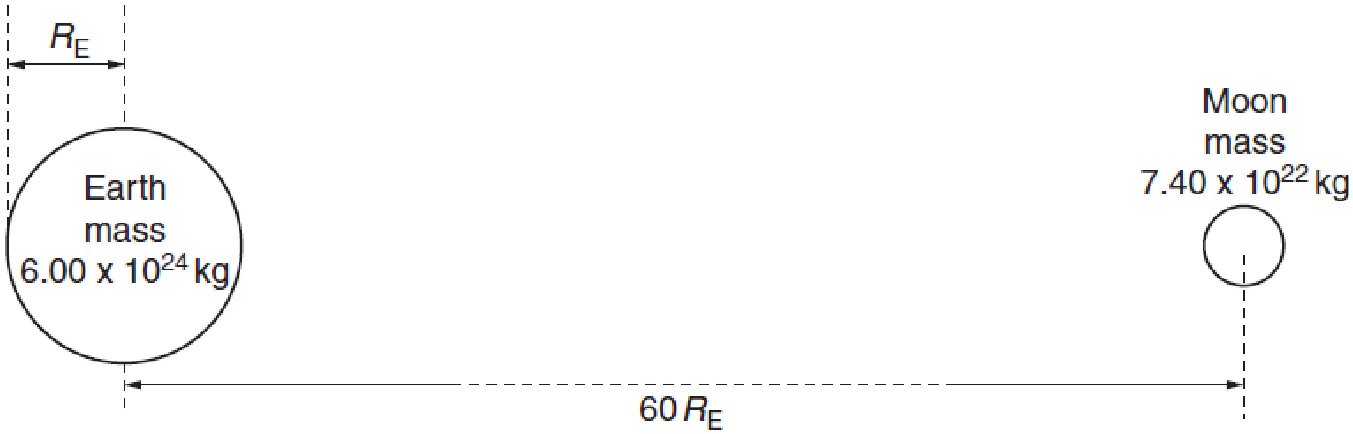

Fig. 1.2 (not to scale)

(i) Explain why there is a point between the Earth and the Moon at which the gravitational field strength is zero.

....

....

.... [2]

(ii) Determine the distance, in terms of R_E, from the centre of the Earth at which the gravitational field strength is zero.

distance = ... R_E [3]

(iii) On the axes of Fig. 1.3, sketch a graph to show the variation of the gravitational field strength with position between the surface of the Earth and the surface of the Moon.

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Fig. 1.3

[3]

{*Nov 10/41 & 42/Q, 1*}

5.

A spherical planet has mass *M* and radius *R*.
The planet may be assumed to be isolated in space and to have its mass concentrated at its centre.
The planet spins on its axis with angular speed ω, as illustrated in Fig. 1.1.

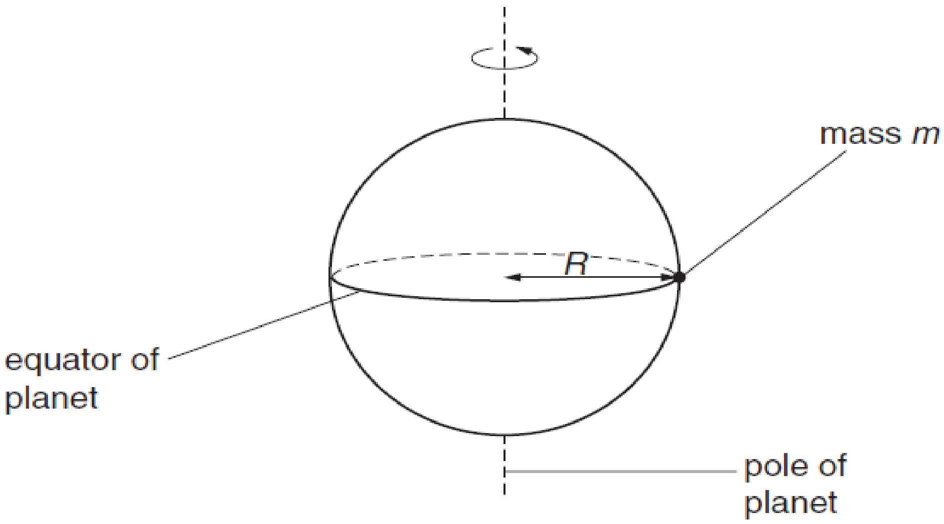

Fig. 1.1

A small object of mass *m* rests on the equator of the planet. The surface of the planet exerts a normal reaction force on the mass.

(a) State formulae, in terms of *M*, *m*, *R* and ω, for

(i) the gravitational force between the planet and the object,

.... [1]

(ii) the centripetal force required for circular motion of the small mass,

.... [1]

(iii) the normal reaction exerted by the planet on the mass.

.... [1]

(b) **(i)** Explain why the normal reaction on the mass will have different values at the equator and at the poles.

....

....

....[2]

(ii) The radius of the planet is 6.4×10^6 m. It completes one revolution in 8.6×10^4 s. Calculate the magnitude of the centripetal acceleration at

1. the equator,

acceleration = ...m s^{-2} [2]

2. one of the poles.

acceleration = ...m s^{-2} [1]

(c) Suggest two factors that could, in the case of a real planet, cause variations in the acceleration of free fall at its surface.

1. ....

....

2. ....

....

[2]

{*Nov 08/4/Q.1*}

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teacher_786@hotmail.com

6. A binary star consists of two stars that orbit about a fixed point *C*,as shown in Fig.6.1.

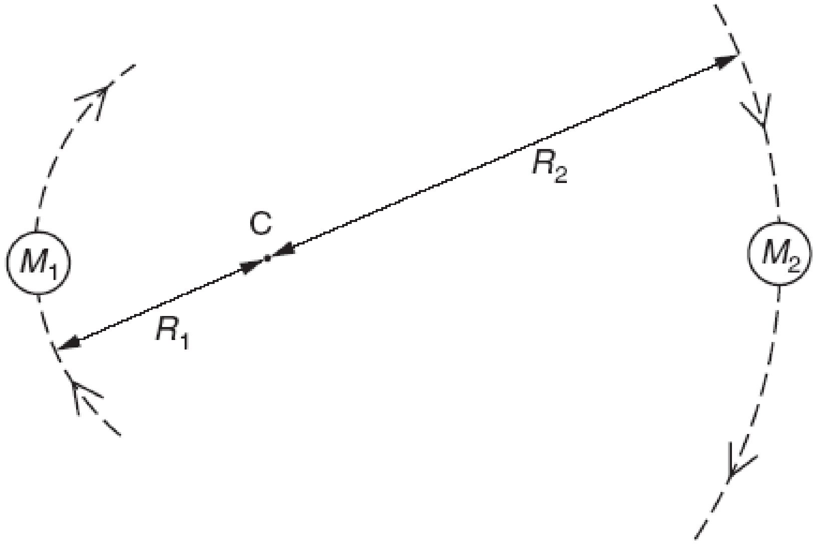

Fig. 6.1

The star of mass M_1 has a circular orbit of radius R_1 and the star of mass M_2 has a circular orbit of radius R_2. Both stars have the same angular speed ω, about C.

(a) State the formula, in terms of G, M_1, M_2, R_1, R_2 and ω for

(i) the gravitational force between the two stars,

....

(ii) the centripetal force on the star of mass M_1.

………………………….. **[2]**

(b) The stars orbit each other in a time of 1.26×10^8 s (4.0 years). Calculate the angular speed ω for each star.

angular speed = ... rad s^{-1} **[2]**

(c) (i) Show that the ratio of the masses of the stars is given by the expression

$$\frac{M_1}{M_2} = \frac{R_2}{R_1}.$$

[2]

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(ii) The ratio M_1 / M_2 is equal to 3.0 and the separation of the stars is 3.2×10^{11} m. Calculate the radii R_1 and R_2.

R_1 = ... m

R_2 = ... m
[2]

(d) (i) By equating the expressions you have given in **(a)** and using the data calculated in **(b)** and **(c)**, determine the mass of one of the stars.

mass of star = ... kg

(ii) State whether the answer in **(i)** is for the more massive or for the less massive star.

.... **[4]**

7. (a) (i) On Fig. 7.1, draw lines to represent the gravitational field outside an isolated uniform sphere.

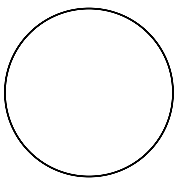

Fig. 7.1

(ii) A second sphere has the same mass but a smaller radius. Suggest what difference, if any, there is between the patterns of field lines for the two spheres.

....

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(b) The Earth may be considered to be a uniform sphere of radius 6380 km with its mass of 5.98×10^{24} kg concentrated at its centre, as illustrated in Fig. 7.2

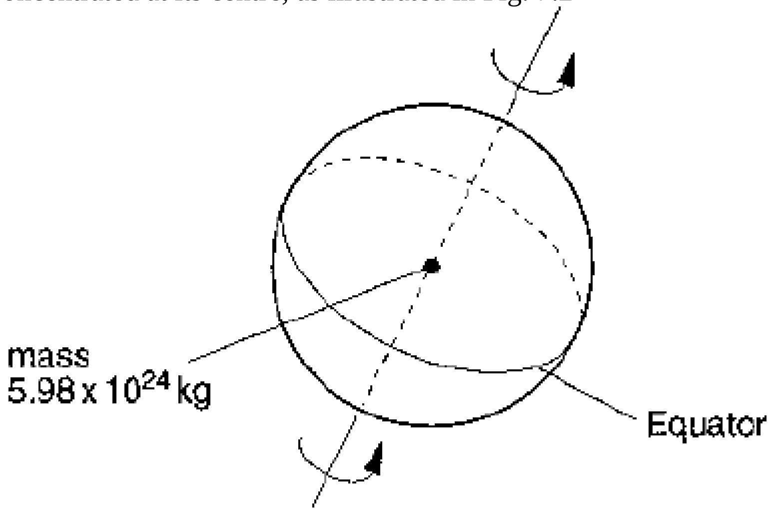

Fig. 7.2

A mass of 1.00 kg on the Equator rotates about the axis of the Earth with a period of 1.00 day (8.64×10^4 s). Calculate, to three significant figures,

(i) the gravitational force F_G of attraction between the mass and the Earth,

F_G = …………………………….. N

(ii) the centripetal force F_C on the 1.00 kg mass,

F_C = ……………………………….. N

(iii) the difference in magnitude of the forces.

difference = ……………………………….. N

(c) By reference to your answers in (b), suggest, with a reason, a value for the acceleration of free fall at the Equator.

………

………

…………………………………………………………………………………………………….... [2]

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8. Fig.84.1 shows part of the orbit of a satellite round the Earth.

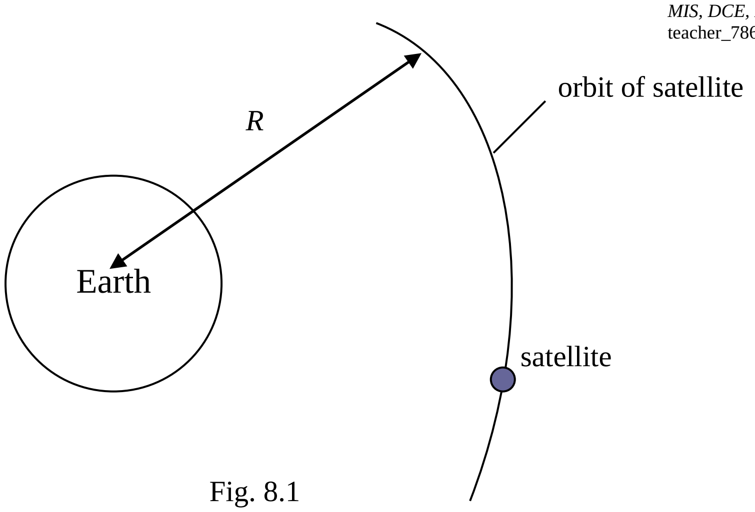

Fig. 8.1

The mass *M* of the Earth is 6.0 x 10^{24} kg. It may be assumed that the gravitational field of the Earth is the same as that of a point mass *M* situated at the centre of the Earth.

(a) On Fig. 8.1 show, by means of an arrow, the direction of the gravitational force on the satellite. [1]

(ii) Explain why the satellite does not move in the direction of the gravitational force.

………

……… [2]

(iii) Show that *v*, the linear speed of the satellite in its orbit of radius *R*, is given by the expression

$$v = \sqrt{GM / R}$$

where *G* is the gravitational constant.

[2]

(b) Two satellite is orbiting the Earth with a radius *R* of 6610 km at a speed v of 7780 ms^{-1}. The satellite is boosted into a higher orbit of radius 6890 km. Show that the speed of the satellite in the new orbit is 7620 m s^{-1}.

[1]

(c) (i) In **(b)**, the satellite, of mass 120 kg, moves from one orbit to another. Using the data in **(c)**, calculate, for this satellite, the change in [6]

1. kinetic energy,

change in kinetic energy = ………………………………… J

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2. gravitational potential energy,

change in potential energy = ………………………………….. J

3. total energy

Change in total energy = ……………………………….. J

(ii) State whether this change in total energy is an increase or a decrease.

……….[1]

9. The Earth may be considered to be a uniform sphere with its mass *M* concentrated at its centre. A satellite of mass *m* orbits the Earth such that the radius of the circular orbit is *r*.

(a) Show that the linear speed *v* of the satellite is given by the expression.

$$v = \sqrt{\left(\frac{GM}{r}\right)}.$$

[2]

(b) For this satellite, write down expressions, in terms of *G*, *M*, *m* and *r*, for

(i) its kinetic energy,

kinetic energy = …………………………. [1]

(ii) its gravitational potential energy

potential energy = ………………………… [1]

(iii) its total energy

total energy = …………………………. [2]

(c) The total energy of the satellite gradually decreases. State and explain the effect of this decrease on

(i) the radius *r* of orbit

....

....

(ii) the speed *v* of the satellite

....

....

[2]

10. A rocket is launched from the surface of the Earth. Fig.10.1 gives data for the speed of the rocket at two heights above the Earth's surface, after the rocket engine has been switched off.

height / m	speed / ms^{-1}
$h_1 = 19.9 \times 10^6$	$v_1 = 5370$
$h_2 = 22.7 \times 10^6$	$v_2 = 5090$

Fig. 10.1

The Earth may be assumed to be a uniform sphere of radius $R = 6.38 \times 10^6$ m, with its mass *M* concentrated at its centre. The rocket, after the engine has been switched off, has mass m.

(a) Write down an expression in terms of

(i) G, M, m, h_1, h_2 and R for the change in gravitational potential energy of the

rocket,..[1]

(ii) m, v_1 and v_2 for the change in kinetic energy of the

rocket..[1]

(b) Using the expressions in (a), determine a value for the mass *M* of the Earth.

M :…………………………………………….. [3]

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